AP Calculus AB
Review of Limits and Continuity

Name Key

Find each limit without a calculator

1. $\lim_{x\to 4}\dfrac{\left(\frac{1}{\sqrt{x}}-\frac{1}{2}\right)}{(x-4)}$ $2\sqrt{x}$ / $2\sqrt{x}$

$\lim_{x\to 4}\dfrac{(2-\sqrt{x})(2+\sqrt{x})}{((x-4)2\sqrt{x})(2+\sqrt{x})}$

$\lim_{x\to 4}\dfrac{4-x}{(x-4)\cdot 2\sqrt{x}(2+\sqrt{x})}$ $\quad \dfrac{-1}{2\sqrt{4}(2+\sqrt{4})}=\dfrac{-1}{16}$

$\lim_{x\to 4}\dfrac{-\cancel{(x-4)}}{\cancel{(x-4)}2\sqrt{x}(2+\sqrt{x})}$

2. $\lim_{x\to 3^-}[[4x-1]]$

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3. $\lim_{x\to\infty}\dfrac{\sqrt{2x^2+3}}{4x+5}$

$\lim_{x\to\infty}\sqrt{\dfrac{2x^2+3}{(4x+5)^2}}$ $\quad \sqrt{\dfrac{1}{8}}$

$\lim_{x\to\infty}\sqrt{\dfrac{2x^2+3}{16x^2\ldots}}$ $\quad \dfrac{1}{2\sqrt{2}}=\dfrac{\sqrt{2}}{4}$

4. $\lim_{x\to 3}\dfrac{x^3-5x-12}{x-3}$

3	1	0	-5	-12
		3	9	12
	1	3	4	0

$\lim_{x\to 3} x^2+3x+4$

$(3)^2+3(3)+4$

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5. $\lim_{x\to\infty}\dfrac{4^{\frac{1}{x}}+2}{5^{\frac{1}{x}}+3}$

$\dfrac{4^{\frac{1}{\infty}}+2}{5^{\frac{1}{\infty}}+3}$

$\dfrac{4^0+2}{5^0+3}=\dfrac{3}{4}$

6. $\lim_{x\to 4}\dfrac{x^2-3x+5}{x-2}$

$\dfrac{(4)^2-3(4)+5}{(4)-2}$

$\dfrac{16-12+5}{2}$

$\dfrac{9}{2}$

7. $\lim_{x\to 3}\dfrac{x^2-6x+8}{x-3}$ $\quad \dfrac{-1}{0}$

$\lim_{x\to 3^-}\dfrac{x^2-6x+8}{x-3}$ $\quad \dfrac{(-)}{(-)}$ ∞

$\lim_{x\to 3^+}\dfrac{x^2-6x+8}{x-3}$ $\quad \dfrac{(-)}{(+)}$ $-\infty$

dne

8. $\lim_{x\to -2^+}\dfrac{x-4}{x+2}$ $\quad \dfrac{-6}{0}$

$\dfrac{(-)}{(+)}$

$-\infty$

9. $\lim_{x\to -\infty}\dfrac{4x^2+x^3-5x-12}{-2x^3-4x-3}$

$-\dfrac{1}{2}$

10. $\lim_{x\to 7^-}\sqrt{x-7}$

dne

11. Use the Intermediate Value Theorem to show that $f(x) = x^3 + x^2 - 5$ has at least one root on the interval [1,2]. Find the root(s). (You may use a calculator)

$f(1) = -3$
$f(2) = 7$

By IVT, since $-3 < f(c) = 0 < 7$ there exists some value c such that $1 < c < 2$

12. Does the IVT guarantee a root for the function $f(x) = \frac{1}{x}$ on [-2,1]?

no, $f(x) = \frac{1}{x}$ is not continuous on the interval from [-2,1]

13. Classify the following functions as continuous or not continuous. If it is not continuous, state the discontinuity type and whether or not it is removable.

a) $f(x) = \frac{x^2 - 4}{x^2 - 3x + 2} = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} = \frac{x+2}{x-1}$

hole: (2,4)
↳ point discontinuity (removable)
VA: $x = 1$
↳ infinite discontinuity (non-removable)

b) $f(x) = \frac{5x - 2}{x^2 + 1}$ f(x) is continuous

c)
$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

$x = 0$:

$f(0) = 3$ $\lim_{x \to 0} f(x) = \text{dne}$

$\lim_{x \to 0^-} f(x) = 0$ $\lim_{x \to 0^+} f(x) = 3$

∴ f(x) is not continuous @ $x = 0$
* jump discontinuity

$x = 3$:

$f(3) = \text{undefined}$ $\lim_{x \to 3} f(x) = 0$

$\lim_{x \to 3^-} f(x) = 0$ $\lim_{x \to 3^+} f(x) = 0$

∴ f(x) is not continuous @ $x = 3$
* point discontinuity

14. Find the values for a and b that make f(x) continuous and then justify continuity.

$$f(x)=\begin{cases}2x^2+3 & x\le -2\\ ax+b & -2<x\le 3\\ -6+\sqrt{x+1} & x>3\end{cases}$$

$2(-2)^2+3 = a(-2)+b$

$11 = -2a+b \rightarrow b = 11+2a$

$a(3)+b = -6+\sqrt{(3)+1}$

$3a+b = -6+2$

$3a+b=-4$

$3a+[11+2a] = -4$

$5a+11=-4$

$a=-3 \quad b=5$

$$f(x)=\begin{cases}2x^2+3 & x\le -2\\ -3a+5 & -2<x\le 3\\ -6+\sqrt{x+1} & x>3\end{cases}$$

$x=-2$:

$f(-2)=11 \quad \lim_{x\to -2} f(x)=11$

$\lim_{x\to -2^-} f(x)=11 \quad \lim_{x\to -2^+} f(x)=11$

$f(-2)=\lim_{x\to -2} f(x)$

$\therefore f(x)$ is cont @ $x=-2$

$x=3$:

$f(3)=-4 \quad \lim_{x\to 3} f(x)=-4$

$\lim_{x\to 3^-} f(x)=-4 \quad \lim_{x\to 3^+} f(x)=-4$

$f(3)=\lim_{x\to 3} f(x)$

$\therefore f(x)$ is cont @ $x=3$

15. Define f (4) so that $f(x)=\dfrac{\sqrt{x}-2}{x-4}$ is continuous at x = 4.

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$$

$$\lim_{x\to 4}\frac{\cancel{\sqrt{x}-2}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}$$

$$\lim_{x\to 4}\frac{1}{\sqrt{x}+2}=\frac{1}{4}$$

hole: $(4, \frac{1}{4})$

$$g(x)=\begin{cases}\dfrac{\sqrt{x}-2}{x-4} & x\ne 4\\ \frac{1}{4} & x=4\end{cases}$$

16. Find the vertical and horizontal asymptotes of the graph of $f(x)=\dfrac{5x^2+3x-2}{x^2-4} = \dfrac{(5x-2)(x+1)}{(x+2)(x-2)}$

VA: $x=-2$

$x=2$

HA: $\lim_{x\to\infty} f(x)=5$

$\lim_{x\to -\infty} f(x)=5$

$y=5$

17. Where does the graph of $f(x)=x^2-5x+1$ have a tangent line parallel to the graph of the line by the equation $3x+6y=36$?

$$\lim_{h\to 0}\frac{(x+h)^2-5(x+h)+1-(x^2-5x+1)}{h}$$

$$\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{5x}-5h\cancel{+1}\cancel{-x^2}\cancel{+5x}\cancel{-1}}{h}$$

$$\lim_{h\to 0}\frac{2x\cancel{h}+h^2-5\cancel{h}}{\cancel{h}}$$

$$\lim_{h\to 0} 2x+h-5$$

$f'(x)=2x-5$

$3x+6y=36$

$m=-\frac{1}{2}$

$2x-5=-\frac{1}{2}$

$2x=\frac{9}{2}$

$x=\frac{9}{4}$

18. Use the definition of derivative to find $f'(x)$ if $f(x)=\sqrt{3-5x}$

$$\lim_{h\to 0} \frac{\sqrt{3-5(x+h)} - \sqrt{3-5x}}{h}$$

$$\lim_{h\to 0} \frac{\left(\sqrt{3-5x-5h} - \sqrt{3-5x}\right)}{h}\frac{\left(\sqrt{3-5x-5h} + \sqrt{3-5x}\right)}{\left(\sqrt{3-5x-5h} + \sqrt{3-5x}\right)}$$

$$\lim_{h\to 0} \frac{3-5x-5h + (3+5x)}{h\left(\sqrt{3-5x-5h} + \sqrt{3-5x}\right)}$$

$$\lim_{h\to 0} \frac{-5\cancel{h}}{\cancel{h}\left(\sqrt{3-5x-5h} + \sqrt{3-5x}\right)}$$

$$\lim_{h\to 0} \frac{-5}{\sqrt{3-5x-5h} + \sqrt{3-5x}}$$

$$\frac{-5}{2\sqrt{3-5x}}$$

19. Find an equation of the line tangent to the curve $y=x^3-2x$ at the point (2,4)

$$\lim_{h\to 0} \frac{(x+h)^3 - 2(x+h) - (x^3-2x)}{h}$$

$$\lim_{h\to 0} \frac{(x+h)(x^2+2xh+h^2) - 2x - 2h - x^3 + 2x}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^3}+2x^2h+xh^2+hx^2+2xh^2+h^3 \cancel{-2x} - 2h \cancel{-x^3} + \cancel{2x}}{h}$$

$$\lim_{h\to 0} \frac{3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}\,2} - 2\cancel{h}}{\cancel{h}}$$

$$\lim_{h\to 0} 3x^2 + 3xh + h^2 - 2$$

$$y' = 3x^2 - 2$$

$$m_{tan} = 3(2)^2 - 2$$
$$= 10$$

equation:

$$y-4 = 10(x-2)$$

OR

$$\lim_{x\to 2} \frac{x^3-2x-4}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -2 & -4 \\ & & 2 & 4 & 4 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

$$\lim_{x\to 2} x^2+2x+2$$

$$10$$

equation:

$$y-4 = 10(x-2)$$